Souksomvang (2021). Souphanouvong Journal of Multidisciplinary Research and Development
ISSN 2521-0653. Volume 7. Issue 1. Jan – Jun 2021. Page 1-10.

Mathematical Model for Population Growth. Case Study: Ethiopia

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Article Info:

Submitted: Nov 10, 2020.

Revised: Dec 04, 2020.

Accepted: Jan 18, 2021.

Abstract

The purpose of this paper is to use mathematical models to predict the population growth of Ethiopia. Ethiopia is an overpopulated country in Africa next to Nigeria. It shares a border with Eritrea to the North and Northeast, Djibouti and Somalia to the East, Sudan and South Sudan to the West, and Kenya to the South. The Malthus's and the logistic growth models are applied to model the population growth of Ethiopia using data from 1980 to 2020. The data used was collected from International Data Base (IDB). We also use the least square method to compute the best population growth rate of Malthus's model. The Malthus's population model predicted a growth rate of 2.9% per year while the logistic growth model predicted the carrying capacity of 51433790163028 and growth rate of 2.9% per annum. The growth rate for both models match well with the growth rate estimated by International Data Base for the past four years. The Mean Absolute Percentage Error is computed as 0.62% for Malthus's population model and 0.64% for logistic growth Model. This showed that the Malthus's population model seems to fit the original data the best among the models we tried.

Keywords

Malthus's population model, Logistic growth Model, Least Square Method, Population Growth rate, Mean Absolute Percentage Error.

1. Introduction

Ethiopia is an overpopulated country in Africa next to Nigeria, and the growth in resources has not been keeping pace with the growth in population. So, the increasing trend in population is a great threat to the nation. Population projections can be used for a number of purposes. Perhaps the most important use of population projections is in the role they can play as a rational basis for decision-making. Changes in population size and composition have many social, economic, environmental, and political implications. For this reason, population projections often serve as a basis for producing other projections (e.g., births, households, families, school enrollment, and labor force).

National population projections, for example, can be used to plan for future Social Security and Medicare obligations (Lee and Tuljapurkar, 1997; Miller, 2001). State projections can be used to determine future water demands (Texas Water Development Board, 1997). Local projections can be used to determine the need for new public schools (Swanson et al., 1998) and to select sites for fire stations (Tayman, Parrott, and Carnevale, 1994). Population projections can be used to forecast the demand for housing (Mason, 1996), the number of people with disabilities (Michaud, George, and Loh, 1996), and the number of sentenced criminals (Oregon Office of Economic Analysis, 2000).

Every government and collective sectors always require accurate idea about the future size of various entities like population, resources, demands and consumptions for their planning activities. To obtain this information, the behavior of the connected variables is analyzed based on the previous data by the statisticians and mathematicians at first, and using the conclusions drawn from the analysis, they make future projections of the aimed at variable. There are enormous concerns about the consequences of human population growth for social, environment and economic development. It is population growth that intensifies all these problems. Mathematical modeling is a broad interdisciplinary science that uses mathematical and computational techniques to model and elucidate the phenomena arising in life sciences. Thus, it is a process of mimicking reality by using the language of mathematics. Many people examine population growth through observation, experimentation or through mathematical modeling. Mathematical models can take many forms, including but not limited to dynamical systems, statistical models and differential equations.

In this paper we model the population growth of Ethiopia by using Malthus and logistic growth models. Furthermore, the paper gives an insight on how to determine the net growth rate of Malthus's population model by using the least square method.

2. Materials and Methods

A research is best understood as a processing of arriving at dependent solutions to the problems through the systematic collection, analysis and interpretation of data. In this paper, secondary classified yearly population data of Ethiopia from (inclusive) were taken from International Data Base (IDB) .The Malthus and logistic growth mathematical models and Least Square Method were applied to compute the projected population values by employing MATLAB. The goodness of the model was assessed using the Mean Absolute Percentage Error.

2.1. The Malthus's Population Model

The simple differential equation

$$\frac{dP}{dt} = \beta P, \qquad \beta > 0 \tag{1}$$

was proposed in 1978 by the English economist Thomas Malthus (T. R. Malthus, 1798) as a basic model for population growth. Here the increase in the population is taken to be proportional to the total number of people, P and β is the constant representing the rate of growth (the difference between the birth rate and death rate). This is the reasonable assumption for the population of bacteria or animal under ideal conditions (unlimited environment, adequate nutrition, absence of predators, and immunity from disease). Suppose we know the population at some given time $t = t_0$, and we are interested for projecting the population, P , at some future time t. The solution of Malthus population model in equation (1) is obtained as follows:

$$\int_{P_0}^{P(t)} \frac{1}{P} dP = \int_{t_0=0}^{t} \beta dt. \tag{2}$$

This gives that

$$P(t) = P_0 e^{\beta t}. \tag{3}$$

This model is often referred to as the exponential law and is widely regarded in the population ecology as the first principle of population of Dynamics. Rearranging equation (3), we can obtain the equation

$$\beta = \frac{\ln(P(t)) - \ln(p_0)}{t}. \tag{4}$$

Thus, the Malthusian parameter β is estimated from equation (4).

2.2. Logistic Growth Model

As population increases in size, the environment's ability to support the population decreases. As the population increases per capita, food availability decreases, waste products may accumulate and birth rates tend to decline while death rates tend to increase. Thus, it seems that reasonable to consider the mathematical model which explicitly incorporates the idea of carrying capacity (limiting value). In 1983 Pierre-Francois Verhulst (P. F. Verhulst, 1838) proposed a model called the logistic model, for population growth. His model does not assume unlimited resources instead it incorporated the idea of the carrying capacity for the population. Thus, the population growth not only on how to depend on the population size but also on how this size is far from its upper limit (maximum supportable population). He modified Malthus's model to make the rate of change

$\frac{dP}{dt}$ of the population P is proportional to the product of the current population P and $\left(1-\frac{P}{K}\right)$

This gives us the logistic differential equation,

$$\frac{dP}{dt} = rP\left(1-\frac{P}{K}\right), \qquad r, K > 0 \tag{5}$$

Where r is the growth rate of population and K is the maximum sustainable population. When the population P is small compared to the parameter K, P^2, is very small, so equation (5) is approximated by $\frac{dP}{dt} = rP$. If the population P is above K, the population decrease, but if below, then it increases.

The Logistic equation has been solved by the method of separation of variables in (JAMES C. ROBINSON, 2004) as follows: Separating the variables gives, $\left(\frac{1}{P}+\frac{1}{K-P}\right)dP = rdt$

We can integrate both sides of equation (6) between the limits corresponding to times $t_0 = 0$ and t

$$\int_{P_0}^{P(t)}\left(\frac{1}{P}+\frac{1}{K-P}\right)dP = \int_{t_0=0}^{t} rdt \tag{6}$$

To give

$$\frac{P(t)(K-P_0)}{P_0(K-P(t))} = e^{rt}$$

Finally, rearrange this gives

$$P(t) = \frac{K}{1+\left(\frac{K}{P_0}-1\right)e^{-rt}} \tag{7}$$

If we take the limit of equation (7) as $t \to \infty$, we get

$$P_{\max} = \lim_{t\to\infty} P = K. \tag{8}$$

He also explained that how the parameters r and K can be estimated from the population $P(t)$ in three different but equally spaced years. If P_0 is the population at time $t = 0$, P_T, that at time $t = T$ and P_{2T} that at time $t = 2T$, then from equation (7), as in (A. Wali, D. Ntubabare, V. Mboniragira, 2011), we can obtain r and K. Implies that

$$\frac{1}{K}(1-e^{-rT}) = \frac{1}{P_T} - \frac{e^{-rT}}{P_0} \tag{9}$$

$$\frac{1}{K}(1-e^{-2rT}) = \frac{1}{P_{2T}} - \frac{e^{-2rT}}{P_0} \tag{10}$$

Dividing equation (10) by the corresponding equation (9) to eliminate K, we get

$$1+e^{-rT} = \frac{\frac{1}{P_{2T}} - \frac{e^{-2rT}}{P_0}}{\frac{1}{p_T} - \frac{e^{-rT}}{P_0}}$$

So that,

$$e^{-rT} = \frac{P_0(P_{2T}-P_T)}{P_{2T}(P_T-P_0)} \tag{11}$$

Equation (11) gives sense if $0 < \frac{P_0(P_{2T}-P_T)}{P_{2T}(P_T-P_0)} < 1$.

From equation (11), the growth rate computed as

$$r = \frac{1}{T}\ln\left(\frac{P_{2T}(P_T-P_0)}{P_0(P_{2T}-P_T)}\right) \tag{12}$$

Plugging this value of r into equation (9), we get the limiting value of P,

$$K = \frac{P_T(P_0P_T - 2P_0P_{2T} + P_TP_{2T})}{P_T^{\,2} - P_0P_{2T}} \tag{13}$$

In Edwards, (Jr., C.H. and Penny, D.E., (1994), the inflection point of logistic function is a point $t = \tau$ at which the second derivative $P''(t) = 0$ changes its sign only on either sides of zero or does not exist. In (Ayele Taye Goshu and Purnachandra Rao Koya, 2013), the inflection point of the logistic function has been solved as follows: For the logistic function in equation (Michaud, J., M. V. George, and S. Loh., (1996), $P'(t)$ and $P''(t)$ are given respectively by

$$P'(t) = rP(t)\left(1 - \frac{P(t)}{K}\right) \quad \text{and}$$

$$P''(t) = rP'(t)\left(1 - \frac{2P(t)}{K}\right).$$

. Here, we observe that $P''(t) = 0$ that is $\left(1 - \frac{2P(t)}{K}\right) = 0$ which gives $P(t) = \frac{K}{2}$ and this implies:

$$\tau = \frac{1}{r}\ln\left(\frac{K}{P_0} - 1\right) \tag{14}$$

Equation (14) is the point of inflection, since $P''(t) = 0$ at the point and also it changes sign only on either sides of zero or does not exist. Hence, for logistic curve, the single point of inflection occurs when the growth reaches half of its final growth $P(\tau) = \frac{K}{2}$ at time given by (14).

2.3. Method of Least Squares

In many branches of Applied Mathematics, Engineering and Sciences, we come across experiments and problems, which involve two or more variables. Here, our focus is to fit the exponential function,

$$P(t) = be^{at} \tag{15}$$

Here, b and a are constants to be determined. For this purpose, we take several sets of observations of time and the corresponding population. The problem is to find the best values for b and a using the observed values of b and t and P. Thus, the principle of Least Square provides unique values to the constants b and a, and hence, suggests a curve of the best fit to the given data. The method of Least Square is probably the most systematic procedure to fit a unique curve through the observed data points.

Suppose that we are given the points $(t_0, P_0), (t_1, P_1), \ . \ . \ . , (t_{n-1}, P_{n-1})$ and we want to fit an exponential

function of the form equation (15). Taking the logarithm of both sides:

$$\ln(P) = \ln(b) + at$$

Then, introduce change of variables:

$$Y = \ln(P), \quad c = \ln(b).$$

This results in a linear relation between the variables Y and t, that

$$Y = c + a.t \tag{16}$$

Thus, the residual e_i is given as $e_i = (Y_i - c - a.t_i), \ i = 0, 1, 2, ..., n-1.$

Now, we have the sum squared deviation,

$$S = \sum_{i=0}^{n-1} e_i^2 = \sum_{i=0}^{n-1} (Y_i - c - at_i)^2 \tag{17}$$

By the Least Squares criterion, the best-fitting curve is that for which the sum of the squared deviations between the estimated and actual values of the dependent variable for the sample data is minimized. Thus,

$$\frac{\partial S}{\partial c} = -2\sum_{i=0}^{n-1}(Y_i - c - at_i) = 0, \ \frac{\partial S}{\partial a} = -2\sum_{i=0}^{n-1}(Y_i - c - at_i)(t_i) = 0 \tag{18}$$

On solving equation (18), we have the system equations,

$$nc + a\sum_{i=0}^{n-1} t_i = \sum_{i=0}^{n-1} Y_i \quad \text{and}$$

$$c\sum_{i=0}^{n-1} t_i + a\sum_{i=0}^{n-1} t_i^2 = \sum_{i=0}^{n-1} t_i Y_i$$

(19)

The system equations (19) is called a normal equation which is used to find the parameter values of c and a that will minimize the least square error (John H. Mathews and Kurtis K. Fink, 2004). After c and a have been found, the parameter b is computed as $b = e^c$. Putting the value of b and a in equation (15), we get the equation of the line that best fits.

2.4. Estimating Optimal Solution to Malthus's Model

In this section, we use least square method to predict the value of β so that the Malthus's population model to have the optimal solution. Thus, method of least square serves as benchmark to compute the best value of parameter β to the Malthus's model. For example, the exponential function in (3) has an option to have different values of β. In other words, for $(n+1)$ number of population, we have (n) values of β computed from equation (4). However, all these values of may not permit the exponential function in nicely approximate the actual population. As a result, it is compulsory to look for a unique and the best value of β, so that the exponential function in (3) approximates the actual population excellently. So, here we propose that $P_0 e^{\beta t} \approx b e^{at}$ which gives that $e^{(\beta - a)t} \approx \frac{b}{P_0}$. After simplification we have $(\beta - a) \approx \frac{1}{t}\ln\left(\frac{b}{P_0}\right)$. Thus, $P_0 e^{\beta t} \approx b e^{at}$ whenever $\beta \approx a$. Now, let we define the absolute deviation (ε) as $\varepsilon = |\beta - a|$

Previously we have stated that method of least square gives unique and best values of b and a to equation (15). Once we have determined the value of a and all values of β from equations (19) and (4), respectively, we need to compute the absolute deviation (ε). Thus, the value at which the absolute deviation is a minimum is the best value of β. Mathematically, first, compute β's values from equation (4) modified as:

$$\beta_i = \frac{\ln(P(t_i)) - \ln(P_0)}{t_i}, \qquad i = 1, 2, ..., n.$$

Then, compute the absolute deviation (ε), i.e.

$$\varepsilon_i = |\beta_i - a|, \quad i = 1, 2, ..., n. \tag{20}$$

Once we have obtained the absolute deviations from equation (20), we choose the most minimum absolute deviation (i.e. $\min\{\varepsilon\}$). Finally, we can determine the time and corresponding population at which the absolute deviation is a minimum. Substituting these values of β and P_0 into equation (3), and it approximates the actual population nicely than others.

Earlier, we have stated that the logistic growth model is approximated by the Malthus's population model when $P << K$. Under this assumption, it is possible to obtain that the solution of the logistic growth model, and for the solution of logistic growth model, we need to use the time t and the corresponding population P at which the absolute deviation in equation (20) is diminishing, so as to estimate the values of the parameters r and K to equation (7). However, as P tends towards K, the above assumption may be rejected.

2.5. Mean Absolute Percentage Error (MAPE)

The mean absolute percentage error is defined as

$$MAPE = \frac{1}{N}\sum\left|\frac{X_t - \tilde{X}_t}{X_t}\right| \cdot 100 \tag{21}$$

Where $X_t, \tilde{X}_t$ and N are the actual, fitted and number of observation of the population respectively. The MAPE is a relative measure which expresses errors as a percentage of the actual data. This is its biggest advantage as it provides an easy and intuitive way of judging the extent, or importance of errors. Lower MAPE values are better because they indicate that smaller percentages' errors are produced by the forecasting model.

3. Results

To estimate the future population of Ethiopia, first we need to determine the constants b and a using the exponential function in (15). Using the actual population of Ethiopia in Table 1 given below, and MATLAB programs, we find the values of c and d from the following system of equations:

$$\begin{cases} 41c + 820a = 737 \\ 820c + 22140a = 14{,}910 \end{cases} \tag{22}$$

So, $c = 17.3972$, $a = 0.0291$ and $b = e^c = 35934198$.

Substituting these values of b and a into equation (15), we obtain:

$$P(t) = 35934198 e^{0.029t} \tag{23}$$

This equation can be used to estimate the value of the dependent variable P given the value of the independent variable t, and it is plotted below, and compared with the actual population of Ethiopia.

Table 2 given below is computed from the equation (20). From this table one can read the minimum absolute deviation $\varepsilon = 2E-06$, and then we can easily understand that time $t = 20(2000)$ and $\beta = 0.0291$. Substituting these values of $\beta = 0.0291$ and $P_0 = 36036457$ into equation (3), the Malthus's population model has the optimal solution,

$$P(t) = 36036457e^{0.0291t} \quad (24)$$

However, it needs to be recognized that this solution is optimal only with respect to the data being used in Table 1. Thus, the value of the population growth rate of Ethiopia is approximately 2.91% per annum with the Malthus's population model. Equation (24) is sketched below, and compared with the data for the growth of the population of Ethiopia. For the logistic growth model, we use $t = T = 20$ and $t = 2T = 40$ with the corresponding values of $P_{20} = 64365225$ and $P_{40} = 114963588$, respectively to predict the parameters r and K. Substituting the values of P_0, P_{20}, and P_{40} into equation (13), we get $K = 51433790163028$. This value of K is the predicted carrying capacity or the limiting value of the population of Ethiopia. From equation (12), we obtain the value of $r = 0.029002146655224 \approx 0.029$. This implies that the predicted rate of Ethiopia's growth is approximately 2.9% per year with the logistic growth model. Finally, substituting the values of P_0, r, and K into equation (7) gives the general solution of the logistic growth model,

$$P(t) = \frac{51433790163028}{1 + 1427270e^{-0.029t}} \quad (25)$$

Based on the data given in Table 1, from equation (14), we obtain the time (year) $t = 488$ at which the growth reaches half of its final growth $P(488) = 25469067884939$. In other words Ethiopian population is growing most rapidly when there are 25469067884939 populations. From equation (21), the mean absolute percentage errors of equations (23), (24) and (25) are computed as 0.77%, 0.62%, and 0.64% respectively.

Table 1: Shows the actual population of Ethiopia from 1980-2020.

Year	Actual Pop	Year	Actual Pop	Year	Actual Pop	Year	Actual Pop	Year	Actual Pop
1980	36036457	1990	47554116	2000	64365225	2010	86042927	2020	114963588
1981	37720093	1991	49447575	2001	66230452	2011	88588758		
1982	39478541	1992	51205192	2002	68167888	2012	91195675		
1983	40673915	1993	52525039	2003	70174826	2013	93877025		
1984	40072730	1994	54030146	2004	72238990	2014	96633458		
1985	40683742	1995	55725462	2005	74354300	2015	99465819		
1986	41642553	1996	57424029	2006	76535275	2016	102374044		
1987	43058596	1997	59076269	2007	78799447	2017	106399924		
1988	44596107	1998	60764879	2008	81135648	2018	109224414		
1989	46093687	1999	62538426	2009	83548430	2019	112078730		

Table 2: Shows the absolute deviation computed using equation (20) with the corresponding values of t, P, β, and a where time (t) is $(1981-2016)$ and P is the corresponding actual population.

t	P	β	a	$\lvert\beta - a\rvert$	t	P	β	a	$\lvert\beta - a\rvert$
1	37720093	0.045662	0.029	0.016662	21	66230452	0.028981	0.029	0.000019
2	39478541	0.045613	0.029	0.016613	22	68167888	0.028988	0.029	0.000012
3	40673915	0.040352	0.029	0.011352	23	70174826	0.028976	0.029	0.000024
4	40072730	0.026541	0.029	0.002459	24	72238990	0.028977	0.029	0.000023
5	40683742	0.024259	0.029	0.004741	25	74354300	0.028972	0.029	0.000028
6	41642553	0.024099	0.029	0.004901	26	76535275	0.028970	0.029	0.000030
7	43058596	0.025433	0.029	0.003567	27	78799447	0.028977	0.029	0.000023
8	44596107	0.026639	0.029	0.002361	28	81135648	0.028985	0.029	0.000015
9	46093687	0.027349	0.029	0.001651	29	83548430	0.028996	0.029	0.000004
10	47554116	0.027734	0.029	0.001266	30	86042927	0.029011	0.029	0.000011
11	49447575	0.028762	0.029	0.000238	31	88588758	0.0290153	0.029	0.000015
12	51205192	0.029276	0.029	0.000276	32	91195675	0.029015	0.029	0.000015
13	52525039	0.028981	0.029	0.000019	33	93877025	0.029014	0.029	0.000014
14	54030146	0.028929	0.029	0.000071	34	96633458	0.029012	0.029	0.000012

15	55725462	0.029060	0.029	0.000060	35	99465819	0.029008	0.029	0.000008
16	57424029	0.029121	0.029	0.000121	36	102374044	0.029003	0.029	0.000003
17	59076269	0.029076	0.029	0.000076	37	106399924	0.029261	0.029	0.000261
18	60764879	0.029027	0.029	0.000027	38	109224414	0.029180	0.029	0.000181
19	62538426	0.029013	0.029	0.000013	39	112078730	0.029094	0.029	0.000094
20	64365225	0.029002	0.029	0.000002	40	114963588	0.029002	0.029	0.000002

Table 3: Shows the projected population of Ethiopia using Malthus's Model, Logistic Model, and Least Square Method.

		Projected Population		
Year	Actual Population	Fitted by Least Square	Malthus's Method	Logistic Method
1980	36036457	35934198	36036457	36036458
1981	37720093	36991548	37096816	37096815
1982	39478541	38080009	38188374	38188373
1983	40673915	39200498	39312052	39312049
1984	40072730	40353957	40468793	40468790
1985	40683742	41541355	41659571	41659566
1986	41642553	42763693	42885387	42885381
1987	43058596	44021997	44147272	44147265
1988	44596107	45317327	45446287	45446279
1989	46093687	46650770	46783526	46783516
1990	47554116	48023450	48160112	48160101
1991	49447575	49436521	49577204	49577191
1992	51205192	50891171	51035993	51035978
1993	52525039	52388623	52537706	52537690
1994	54030146	53930137	54083607	54083588
1995	55725462	55517009	55674995	55674974
1996	57424029	57150574	57313209	57313186
1997	59076269	58832207	58999627	58999601
1998	60764879	60563321	60735668	60735639
1999	62538426	62345372	62522790	62522758
2000	64365225	64179859	64362498	64362462
2001	66230452	66068325	66256338	66256299
2002	68167888	68012359	68205904	68205861
2003	70174826	70013595	70212835	70212788
2004	72238990	72073717	72278819	72278768
2005	74354300	74194457	74405594	74405539
2006	76535275	76377598	76594948	76594888
2007	78799447	78624978	78848724	78848658
2008	81135648	80938486	81168815	81168744
2009	83548430	83320069	83557175	83557098
2010	86042927	85771728	86015811	86015727
2011	88588758	88295526	88546791	88546701
2012	91195675	90893586	91152245	91152147
2013	93877025	93568093	93834362	93834257
2014	96633458	96321296	96595400	96595287
2015	99465819	99155511	99437681	99437559
2016	102374044	102073122	102363594	102363463
2017	106399924	105076582	105375602	105375460
2018	109224414	108168418	108476236	108476083
2019	112078730	111351230	111668105	111667941
2020	114963588	114627694	114953894	114953718

4. Discussion

From Table 1, it can be seen that in 1980 population of Ethiopia was 36036457 and continued to increase till 1983 when it decreased considerably from 40673915 to 40072730 in 1984. This reduction was probably due to famine, starvation, war and some people's migration to other countries. The population began to rise once again from 40072730 in 1984 to 114963588 in 2020. There has been an improvement in the education, agricultural productivity, water and sanitation and health services in the country. There were also early marriages and long time belief that more children one had, one would have a higher social and economic status, have higher work force in their farms and receive better care in old age. All these with other factors had an overall effect on the increase in population.

From figure 1 and 2, we see that the actual population and predicted values computed from Least Square method and Malthus's model respectively, are very close to each other. As we see in Figure 4 for the parameter value $\beta = 0.046$ of Malthus's model, the actual and the predicted population are not matching very well as time (year) increases. This shows that the values of β computed from equation (4) could not be the best value so that the Malthus's model to have the best solution. Thus, equation (20) is better assumption as β has optimal value. Figure 3 shows that the predicted population has a better agreement with actual population. As time (year) increases, logistic model has greater mean absolute percentage error than others. This is may be due to the assumption $P << K$. This is presented in Figure 5.

The population growth rate of Ethiopia according to information in International Data Base (IDB) was approximately 2.9% in 2012 and 2013, 2.89% in 2014 and 2015 which corresponds well with the findings in this research work of a growth rate of approximately 2.9% per annum.

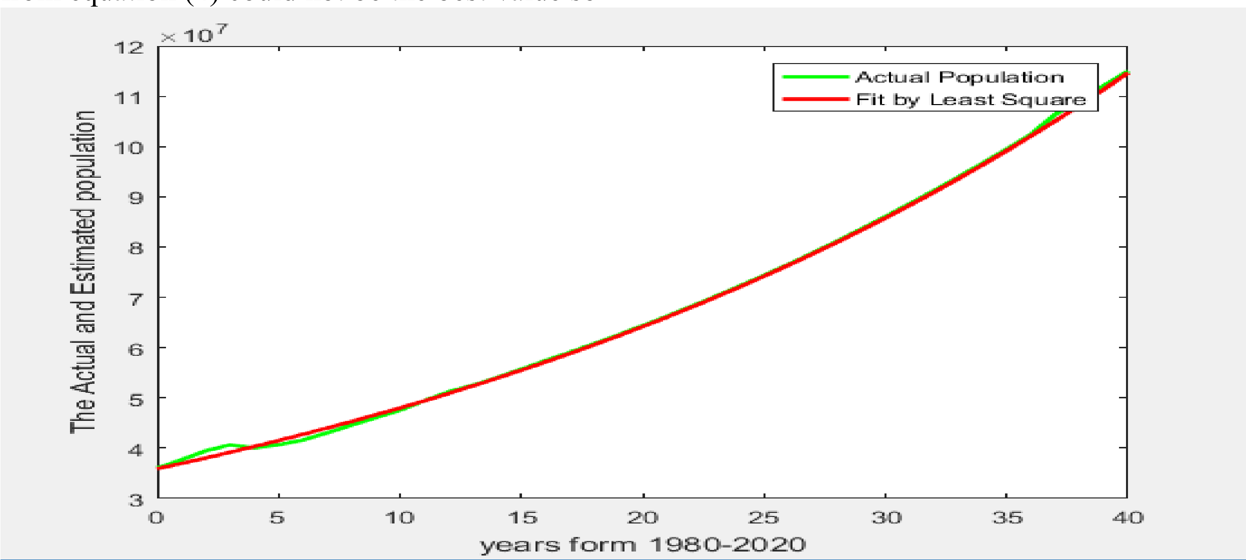

Figure 1: Shows the graph of actual population and the projected population using Least Square Method.

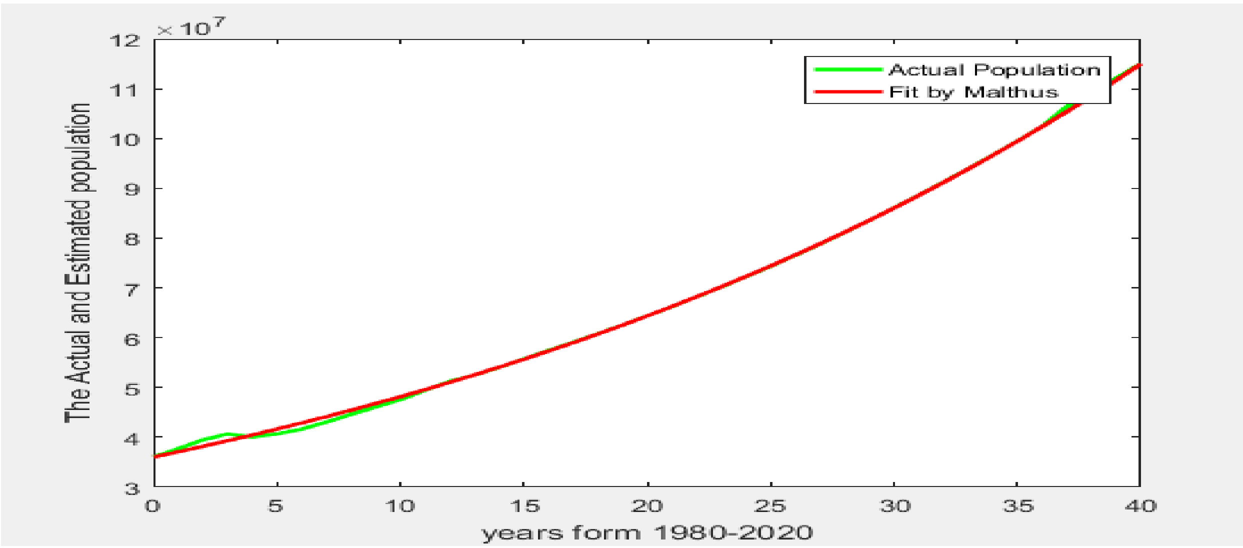

Figure 2: Shows the graph of actual population and the projected population using Malthus's model

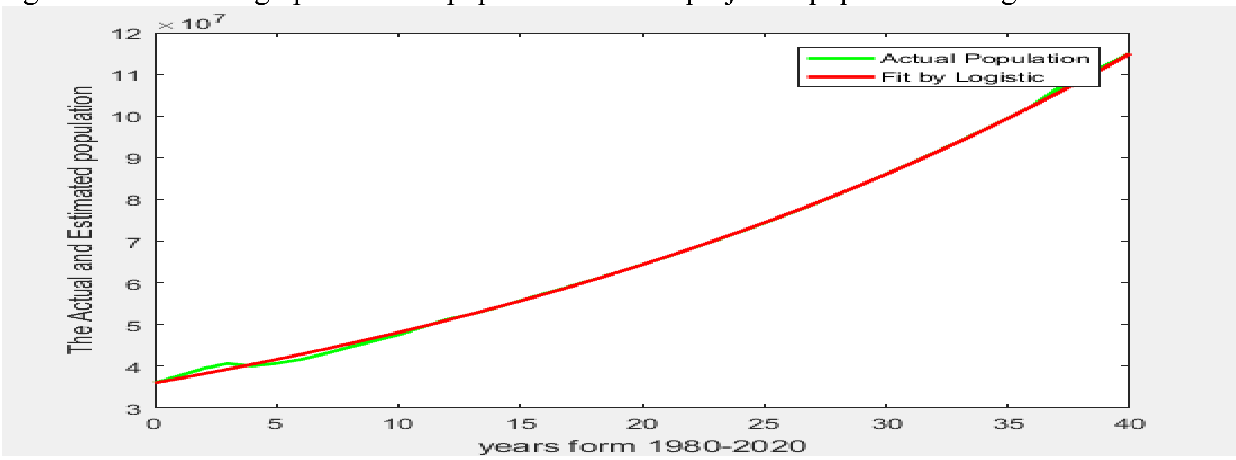

Figure 3: Shows the graph of actual population and the projected population using Logistic model

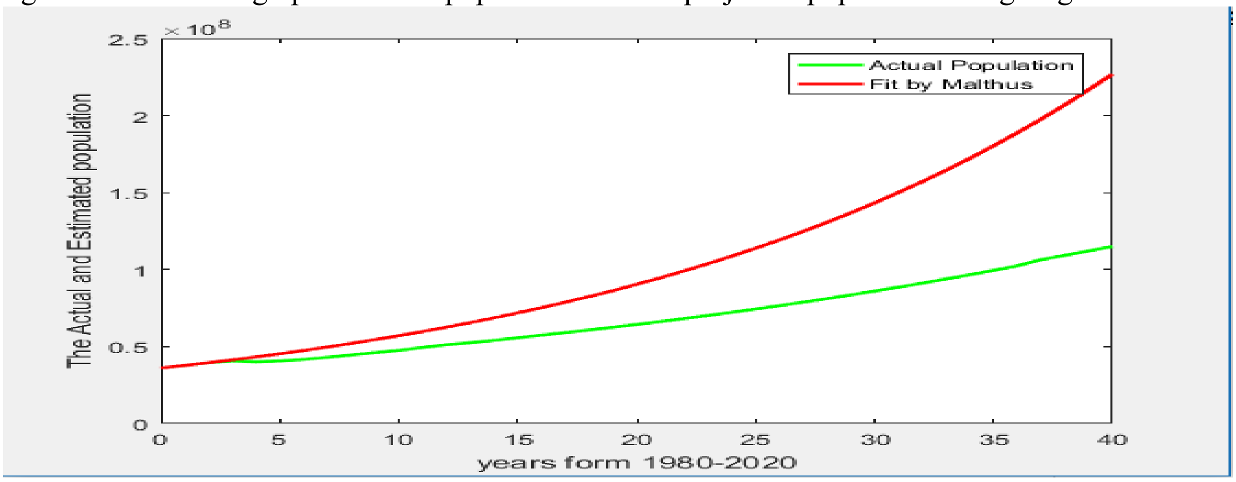

Figure 4: Shows the graph of actual population and the projected population using Malthus's model with $\beta = 0.046$

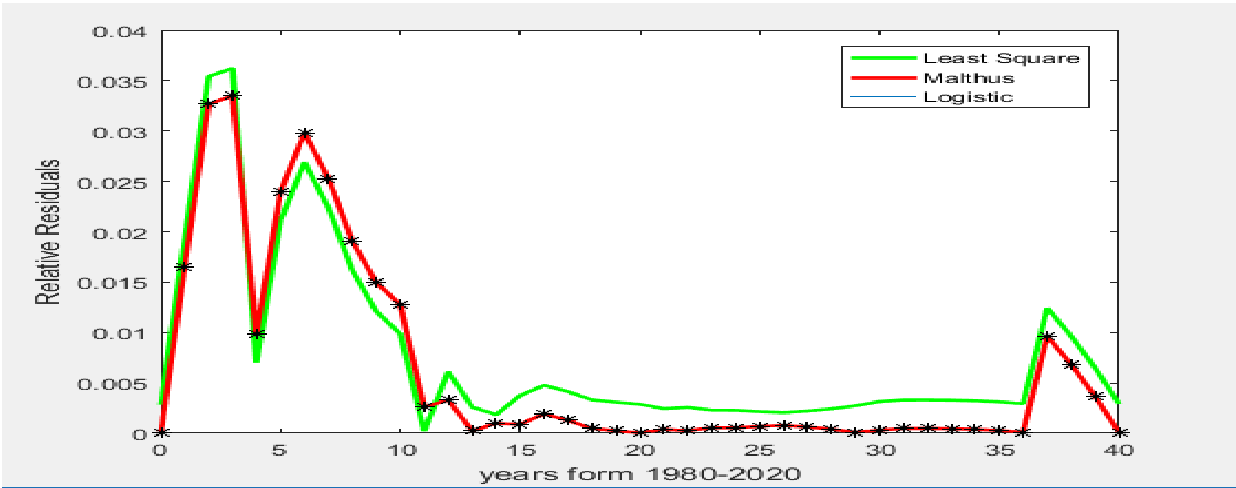

Figure 5: Shows the graph of relative residuals of the projected population using three models.

5. Conclusions

In this study, a mathematical analysis of the future population of Ethiopia is carried out based on ordinary differential equation models which are Malthus's population model and Logistic model. The study applies least square method in order to determine the optimal value of the growth rate of the Malthus's model. The Malthus's population model predicts a growth rate of approximately per year with a MAPE of 0.62%. The Logistic Model on the other hand, predicts a carrying capacity for the population of Ethiopia to be $K = 51433790163028$ while the population growth rate of Ethiopia is approximately 2.9% per annum with a MAPE of 0.64%. Based on this, we can conclude that the Logistic and Malthus's population models seem to fit the original data the best among the models we tried. From Logistic model we also found out that the population of Ethiopia is expected to grow most rapidly when there are 25469067884939 (half of its final growth) populations in the year $t = 488$. MATLAB program based on an algorithm in appendix is used for calculation of future population. Furthermore, the growth rate for both models is approximately 2.9%. As a recommendation, we forward that it is better to find the most appropriate value of the growth rate of Logistic model since it has many values and use the proposed method to find the best value of the growth rate of the Malthus's population model.

6. Conflict of Interest

We certify that there is no conflict of interest with any financial organization regarding the material discussed in the manuscript.

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