# Comparison of Traditional and Copula based Value-at-Risk with Low Volatility Equity Portfolio: Empirical Evidence from Laos Securities Exchange

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**Abstract**

While eleven stocks are currently registered and traded in the Laos Securities Exchange (LSX), risk regarding market volatility is kind of tricky to measure. Value-at-risk (VaR) is a popular statistical measure for the return of financial assets that are normally distributed. However, the distribution of a portfolio's return is uncommonly characterized by the assumption of joint normality that many classical financial theories are assumed. Therefore, the copula is employed to draw out a multivariate probability distribution. This research aims to compare the performance between the traditional and copula based VaR with the two most actively traded but low volatility stocks (BECL and EDL-Gen) registered in the LSX between November 2019 to November 2020. The daily returns were calculated to evaluate the possible marginal distributions that were well describing the behavior of stock's return. The Anderson-Darling tests would be applied to assess a goodness-of-fit between historical time series of daily return and the possible probability distributions which are normal, student-t, log-normal, logistic, triangular, Gumbel, Fréchet, Weibull, generalized beta, and generalized extreme value distributions. The tested results found that the Gumbel distribution, extreme value distributions type-I, was well suit to describing the behavior of both stock's returns. The maximum likelihood estimation method was then used to estimate parameters of the Gumbel distributions. The random samples from a multivariate normal distribution were generated based on variance-covariance matrix of both stocks. The Gaussian copula, which is tail independent and also allows for negative dependence, and correlation matrix of the generated random samples were used to generate stock's returns. Based on equally weighted average of all individual estimated stock's returns held in the portfolio, the portfolio's returns were recalculated in a number of 10,000 times. Then, the portfolio VaRs based on normality assumption and copula were estimated in according with 95%, 97.5% and 99% levels of confidence respectively. The VaR based on Gaussian copula was slightly lower than the normality VaR for all given levels of confidence respectively. This can be implied that the Gaussian copula VaR was not consistently aligned with the conservative portfolio investment where investing in low-risk securities is prioritized. Even though, the portfolio based on low-volatility stocks was formulated, the Gaussian copula VaR was not favorable for risk-averse investors to measure the worse loss depending on the current position.

**Keywords**

Risk Measures, Copula Functions, Value-at-Risk, Probability Distribution, Laos Securities Exchange.

## 1. Introduction

Stock market investment is one of the alternative choices to leverage investors expected return. However, uncertainty of return, so called risk, is the most stressful to general investors. Analyzing stock market risk and establishing the portfolio risk management strategies are an important task of investing. There are several factors affecting the volatility of stock market which may include economic growth, recessions, inflation, interest rates, currency fluctuations, and etc. These factors are unpredictable yet seriously create volatility and risk in the stock market. Interestingly, the volatility does not seem to bother most investors during bull markets. Most investors do not even realize that the portfolio volatility by itself will reduce returns. In addition, as more and more people follow the majority of investors, they will let their emotions cause them to buy more when prices are high and sell after prices have fallen. Another common problem is investors seeking high rates of return concentrate on higher risk stocks. Riskier stocks tend to be smaller and fundamentally weaker than the average stock. The more speculative stocks tend to lead the market up in rallies, but collapse in down markets.

**Table 1**: List companies in the LSX

| Company Name | Issue Name | Foreign Investment Ratio | Occupation Ratio (%) |
|---|---|---|---|
| Banque pour le Commerce Exterieur Lao Public | BCEL | 30 | 10.513 |
| EDL-Generation Public Company | EDL-Gen | 25 | 12.061 |
| Mahathuen Leasing Public Company | MHTL | 100 | 29.290 |
| Lao Asean Leasing Public Company | LALCO | 100 | 20.975 |
| Petroleum Trading Lao Public Company | PTL | 100 | 12.188 |
| Souvanny Home Center Public Company | SVN | 100 | 4.180 |
| LAO World Public Company | LWPC | 100 | 0.922 |
| Lao Cement Public Company | LCC | 100 | 9.853 |
| Vientiane Center Lao Public Company | VCL | 100 | 0.786 |
| Phousy Construction and Development Public Company | PCD | 100 | 0.377 |

**Source:** Laos Securities Exchange (2020) consider the overview of Laos investment scheme, even though the Lao Securities Exchange (LSX) would like to increase the number of listed companies, there are currently eleven companies registered in the stock market (LSX, 2020) where Banque Bour Le Commerce Exterieur Lao Public (BCEL), EDL-Generation Public Company (EDL-Gen) and Mahathuen Leasing Public Company (MHTL) are top 3 of the daily largest volume ranking. In addition, about over half of the stocks are mostly traded by foreign investors. This is the reason that Laos market volatility is likely low as there are a lot of low-frequency trading stocks traded in the market. Even though, the volatility in the Laos stock market is likely rigid, the investors still need to prepare for a protective strategy in speculating the market uncertainty.

**Figure 1:** Historical time series of LSX Index during January 2019 – December 2020

Source: Laos Securities Exchange (2020)

Since it is hardly to accurately predict what factors will cause the market to go up or down, aninvestor needs to manage the effects of systematic risk has on their investment portfolio. In order to implement any risk management strategies, the investors need to measure the risk level of their portfolio. The general risk measure for portfolio investment is a Value-at-Risk (VaR) which the VaR is a statistical technique used to measure the amount of potential loss that could happen in an investment portfolio over a specified period of time. The idea of VaR is not newly discovered because they drew the flamework and also are major components in modern portfolio theory (MPT) as suggested by Markowitz (1952). In other words, the value of VaR gives the probability of losing more than a given amount in a given portfolio. In order to calculate the VaR based on traditional approach (variance-covariance method), a normal distribution in returns is mandatorily required to assume. Moreover, there are two major factors that are needed to be estimated – an expected return and a standard deviation. This method is best suited to risk measurement problems where the distributions are known and reliably estimated. However, this method is unreliable when the sample size is very small.

With the mandatory requirement of a normal distribution in returns, we rarely find in the practical results on the VaR's estimation because we all know that the stock market returns are not generally normally distributed. Instead, we obviously found that they are having fat tails where the extreme events happen more frequently than expected. But if we look at the distribution of stock market returns over different time frames, we then will find that the returns are not even monomodal that they do not just have one peak in the middle of the distribution as predicted by the normal distribution. As results, applying the normality VaR would be bias in estimating the risk level regarding portfolio investment.

Furthermore, the interdependence of returns of two or more assets is generally calculated using the correlation coefficient. However, the correlation works best with normal distributions, while distributions in financial markets are often non-normal in nature. The copula, therefore, has been applied to several areas of finance and risk management, such as option pricing and portfolio VaR, to deal with skewed or asymmetric distributions. The copula is a probability model that represents a multivariate uniform distribution, which examines the association or dependence between many random variables.

With a dynamic tool in modelling a multivariate distribution without the assumption of joint normal distribution, applying the copula with the VaR approach is practically an alternative choice for the portfolio risk measurement in which the copula provides benefit on a multivariate joint distribution merging the marginal distribution and the dependence between variables. There are several works applying copula with the VaR approach but measuring the dynamic risk level as copula VaR in a low frequency trading market is limited in the literatures. Some applications of copula in VaR estimation are Cherubini and Luciano (2001) who applied Archimedean copula family and the historical empirical distribution to estimate the marginal distribution, and Fortin and Kuzmics (2002) who used a linear combination of copula to estimate the portfolio's VaR. In addition, Khruachalee and Bodhisuwan (2019) estimated copula VaR and copula conditional VaR for the portfolio of SET, IDX and KLCI stock indices. This research aims to compare the traditional and copula-based value-at-risk with low volatility equity portfolio which is represented by the two most actively traded stocks (BECL and EDL-Gen) registered in the LSX. These two stocks can describe the overall movement of the Laos stock market transaction during November 2019 to November 2020 where the outbreak of Covid-19 still exists.

## 2. Materials and Methods

### 2.1 Data

The historical time series of the daily closed price of Banque Pour Le Commerce Exterieur Lao Public (BCEL) and EDL-Generation Public Company (EDL-Gen), which they were actively traded stocks from November 2019 to November 2020, were collected from the databases of Investors.com's website. In addition, the historical time series of the daily closed price of each stock was transformed becoming a historical time series of daily return by applying the log transformation of return.

$$R_{i,t} = \ln\left(\frac{P_{i,t+1}}{P_{i,t}}\right)$$

where $R_{i,t}$ denotes the rate of return of stock $i$ at time $t$.

$P_{i,t+1}$ denotes the closed price of stock $i$ at time $t+1$

$P_{i,t}$ denotes the closed price of stock $i$ at time $t$

### 2.2 The Portfolio Investment Theory

According to Markowitz (1952), the portfolio's return ($R_p$) is determined by the weighted average of all individual expected returns held in the portfolio.

$$R_p = w_1R_1 + w_2R_2 + \cdots \qquad = \sum_{i=1}^{N} w_iR_i$$

where $w_i$ denotes the percentage composition of a particular holding assets in a portfolio.

$R_i$ denotes the expected rate of return of each individual asset.

Accordingly, expected return of portfolio ( $E(R_p)$ ) is

$$E(R_p) = \sum_{i=1}^{N} w_i\mu_i$$

where $\mu_i$ denotes the mean return of each individual asset.

In addition, the portfolio's variance ( $V(R_p)$ is

$$V(R_p) = \sigma_p^2 = \sum_{i=1}^{N} w_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N} w_iw_j\sigma_{ij}$$

$$= \sum_{i=1}^{N} w_i^2\sigma_i^2 + 2\sum_{i=1}^{N}\sum_{j>i}^{N} w_iw_j\sigma_{ij}$$

where $\sigma_p^2$ denotes the portfolio's variance.

$\sigma_i^2$ denotes the individual stock return variance.

### 2.3 Analytical Approach of VaR

VaR is a common tool for measuring and managing risk in the financial industry. Since the VaR is also defined as the statistical measurement of the worst loss depends on the current position, the predetermined probability distribution that certain loss will be larger than the value of VaR is then defined as follow.

$$P\left(Loss > VaR\right) \leq 1-\alpha$$

where $\alpha$ denotes the confidence level.

$Loss$ denotes the loss level.

With the probability distribution of the portfolio's return, the VaR can be derived by

$$P(R_p \leq R_p^-) = \int_{-\infty}^{R_p^-} f(R_p)\,dR_p = 1-\alpha$$

where $R_p^-$ represents the worst possible loss realization of the investment portfolio which can be defined as the quantile of the distribution which the cut-off value with the predetermined probability of being exceeded.

### 2.4 The Mathematical of Copula

Suppose the marginal distributions of a random vector $(R_1, R_2, \ldots$ are continuous which marginal cumulative distribution function (cdf) is $F_i(r_i) = P(R_i \leq r_i)$. With the integration of each component, the random vector

$(U_1, U_2, \ldots \quad \ldots$

has uniformly distributed marginals.

The copula of $(R_1, R_2, \ldots$ is well defined by the joint cumulative distribution function of $(U_1, U_2, \ldots$ as

$C(u_1, \ldots \quad \ldots$

The contribution of this expression is to generate a pseudo-random sample from the multivariate probability distribution which required sample can be illustrated as

$(R_1, R_2, \ldots \quad \ldots$

If $F_i$ are assumed to be continuous distribution, the inversion of $F_i^{-1}$ is uncomplicated which can be expressed as

$C(u_1, \ldots \quad \ldots$

### 2.5 Fitting the Distributions of Stock Return

Determining a marginal probability distribution of each stock return, the maximum likelihood estimation method will be used to estimate the parameters of competed probability distributions which is well describing the time series of stock return where the competed distribution consists of normal, student-t, log-normal, logistic, triangular, Gumbel, Fréchet, Weibull, generalized beta, and generalized extreme value distributions. These distributions were widely discussed in the literatures for describing the stock return in several markets. Then, the Anderson-Darling tests will be employed to evaluate the possible probability distributions that can be best described the behavior of each stock return.

## 3. Results

### 3.1 Fitted Distribution of Stock's Return

By applying the historical time series of daily returns of BCEL and EDL-Gen with the function *"fitdistrplus"* by Delignette-Muller et al., (2008) in R programing language (R Core Team, 2020), we can estimate the parameters of the competed probability distributions that is well describing each stock's return. Then, the goodness-of-fit results provided by the Anderson-Darling tests will be used to evaluate the most fitted distribution. With the empirical results, we can indicate that the Gumbel distribution outperformed in describing the return of both stocks. Therefore, the Gumbel distributions

BCEL ~ $l(-0.008696, 0.025421)$

EDL-Gen ~ $l(-0.010894, 0.025462)$

were used in modelling the multivariate distribution. The Gaussian Copula, which is tail independent and also allows for negative dependence, will be employed to bridge the dependent sturucture.

**Table 2:** The estimated parameter $\hat{\alpha}$ and $\hat{\beta}$ which determine the location and scale of the Gumbel distribution of each stock return.

| Symbol | Parameter | |
|---|---|---|
| | Location ( $\alpha$ ) | Scale ( $\beta$ ) |
| BECL | -0.008696 | 0.025421 |
| EDL-Gen | -0.010894 | 0.025462 |

Table 2 indicates the parameter $\hat{\alpha}$ and $\hat{\beta}$ of the Gumbel distribution that best describe each stock's return where the $\hat{\alpha}$ and $\hat{\beta}$ are location and scale parameters respectively. In addition, we also applied the "*plotdist"* function by Delignette-Muller et al., (2008) in R programing language (R Core Team, 2020) to provide the plots of empirical and theoretical density for each daily stock return as illustrated in **Figure 2-3.**

Based on the results of goodness of fit test, it can be surely confirmed that the stock's return does not follow normal distribution as mentioned in various classical financial theories. In addition, each stock returns do not just have only one peak in the middle of the distribution. Thus, the normal distribution may not appropriate in predicting the possible value. As results, estimating the copula VaR with the appropriated marginal distribution would lead to the right track of investors in determining the portfolio's risk level.

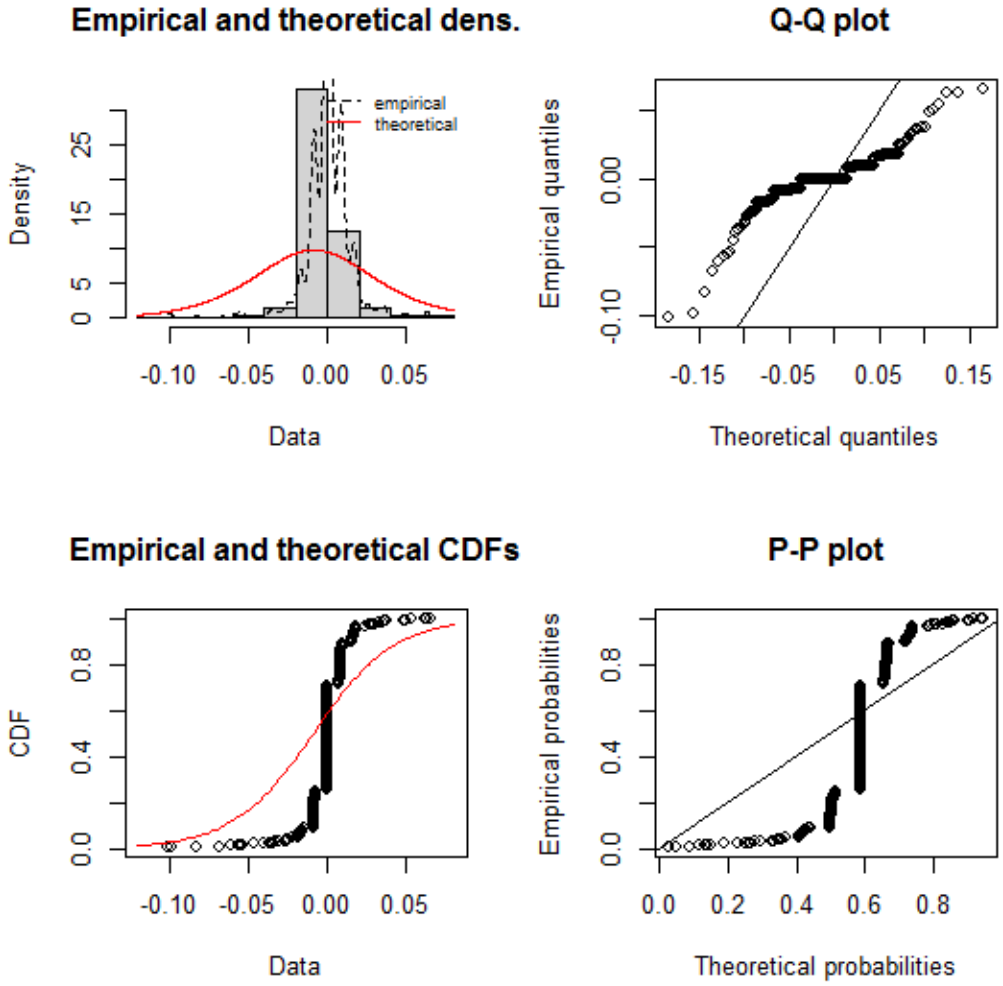


**Figure 2:** Empirical and theoretical density of BECL's return

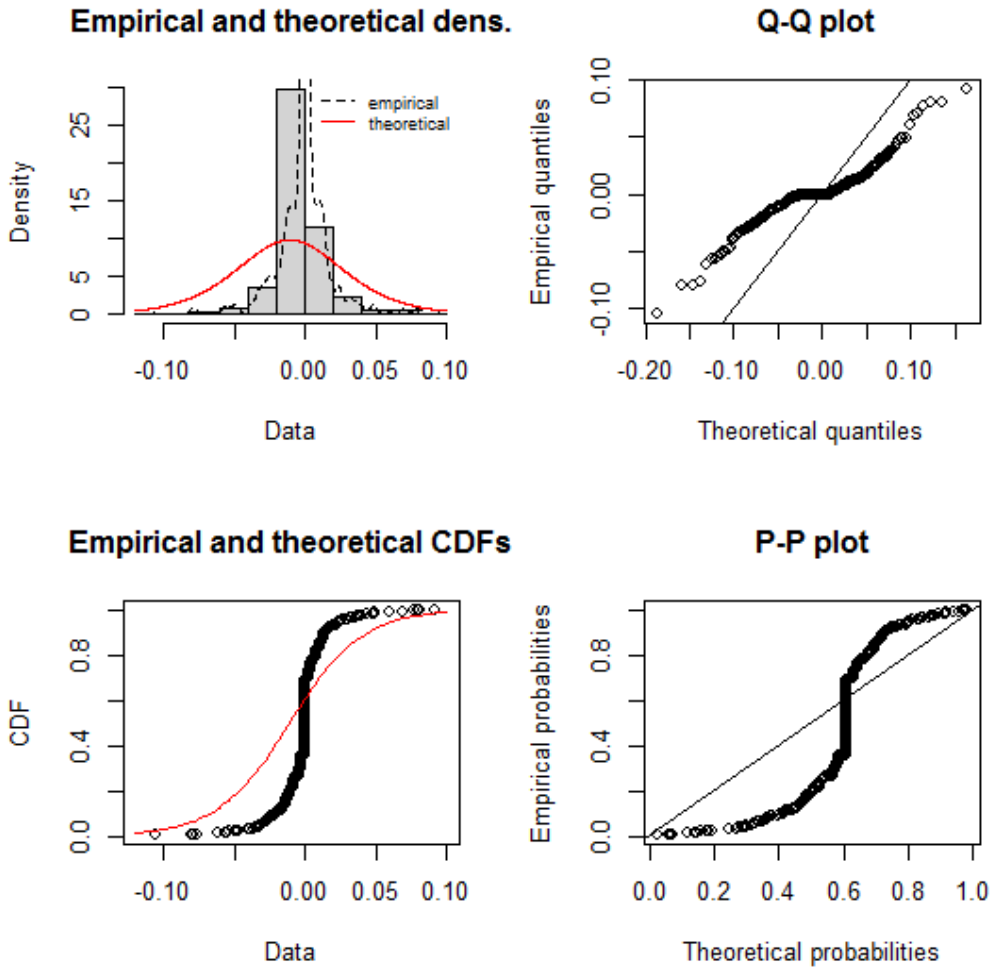


**Figure 3:** Empirical and Theoretical density of EDL-Gen's return

### 3.2 The Simulation for VaR

By randomly select 1,000 random variables from a standard normal distribution where each variable mutually shares their relationship through the variance-covariance matrix, the vector of cumulative probability $(u)$ based on the random sample will be generated by the application of function "*pnorm*" of Venables and Ripley (2002) in R package.

With the implementation of Gaussian copula and the correlation matrix ( $\rho$ ) between the daily return of BCEL and EDL-Gen, the simulated rate of return will be generated based on the inversion of joint cumulative distribution function of the Gumbel distribution.

$$C(u_{BCEL}, u_{EDL-Gen}; \rho) = \Phi_{\rho}(\Phi^{-1}(u_{BCEL}), \Phi^{-1}(u_{EDL-Gen}))$$

where $u_{BCEL} = F_{BCEL}(R_{BCEL})$

$u_{EDL-Gen} = F_{EDL-Gen}(R_{EDL-Gen})$

$\Phi_{\rho}$ is a normal cumulative joint distribution function of a multivariate Gaussian distribution with mean vector zero and $n \times n$ correlation matrix $\rho$.

$\Phi^{-1}$ is a probability density function whose values are inversed from normal cumulative distribution.

$\rho$ is the correlation matrix between variable $\Phi^{-1}(R_{BCEL})$ and $\Phi^{-1}(R_{EDL-Gen})$.

Then, the portfolio's return ( $R_p$ ) based on the equally weighted average of all individual expected returns ( $w_i = 0.5$ ) are recalculated in a number of 10,000 times. As results, there are 10,000 simulated value of the portfolio's return.

The function *"fitdistrplus"* by Delignette-Muller et al., (2008) in R programing was implemented again to estimate the best fitted distribution of portfolio's return which the Gumbel distribution was also well describing the portfolio's return.

$$R_p \sim \quad l(-0.007088, 0.016702)$$

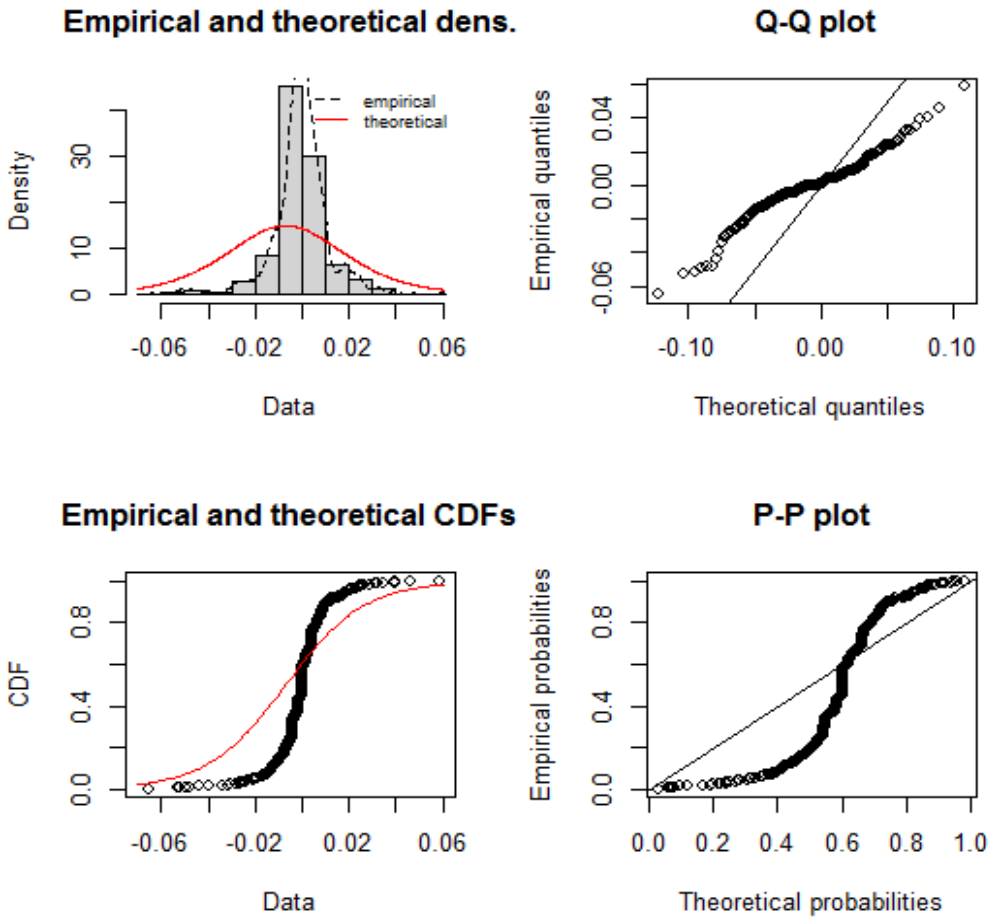


**Figure 4:** Empirical and Theoretical density of portfolio's return ( $R_p$ )

To estimate the portfolio's VaR based on copula approach in according with 95%, 97.5% and 99% levels of confidence respectively, the probability distribution of portfolio's return ( $R_p$ ) was integrated based on the following expression.

$$P(R_p \le VaR_p) = \int_{-\infty}^{VaR_p} f(R_p)\,dR_p = 1-\alpha$$

$$\text{where } f(R_p) = \frac{1}{0.016702} e^{\left(\frac{r_p+0.007088}{0.016702}+e^{-\left(\frac{r_p+0.007088}{0.016702}\right)}\right)}$$

**Table 3:** Risk comparison of the portfolio consisted of low volatility equities (BECL and EDL-Gen) based on various assumptions of VaR

| Confidence Level | Normality VaR | Gaussian Copula VaR |
|---|---|---|
| 99.0 % | 1.7351% | 1.6493% |
| 97.5 % | 1.5125% | 1.3979% |
| 95.0 % | 1.1203% | 0.9897% |

Table 3 indicated that the Gaussian copula VaRs of low volatility equity portfolio were slightly lower than the normality VaR for all given levels of confidence. This can be implied that the normality VaR of low volatility equity portfolio was consistently aligned with a conservative portfolio investment where investing in low-risk securities is prioritized such as blue-ship stock, high dividend stock and big cap stock. Even though, the portfolio based on low-volatility equity was formulated, the Gaussian copula VaR was not favorable for risk-averse to measure the worse loss.

## 4. Discussion

According to the empirical results, we can imply that the normality VaR based on low-volatility equity will be used by risk averse investors which aligned with the study of Khanthavit, A. (2007) that studies the performance of copula VaR and normality VaR of fixed income portfolio. He found that the normality VaR of fixed income portfolio

will make the fixed income portfolio management more conservative than using copula VaR. In addition, the study of Khochorn, K. and Chaiwat, T. (2014) found that copula VaR and Normality VaR of bond portfolio are very similar which they can be used to measure the level of risk more effectively. Therefore, the low volatility equity portfolio management using copula VaR with is aggressive than the use of normality VaR.

## 5. Conclusion

There are several works applying copula with the VaR approach but measuring the dynamic risk level as copula VaR in a low frequency trading market is limited in the literatures. The Laos Securities Exchange (LSX) is kind of tricky to measure risk regarding market volatility because most of the stock are traded by foreign investors. This is the major reason that the Laos market volatility is likely low as there are a lot of low-frequency trading stocks traded in the market. Even though, the Banque Pour Le Commerce Exterieur Lao Public (BCEL) and EDL-Generation Public Company (EDL-Gen) are mostly trade in the market, they will be used to represent a low volatility equity portfolio. This paper found that the application of copula in modelling a multivariate distribution regardless of the assumption of joint normality with the VaR approach to determine the portfolio's risk level of low volatility equity would favor for the investors who are risk lover. As the estimated VaRs based on normality assumption were slightly higher than the Gaussian copula VaR for all given levels of confidence, the normality VaR would be considered as a conservative portfolio investment strategy for risk-averse. However, the conservative portfolio investment strategy does not indicate the best performance of portfolio management. On another hand, the accurately estimated value of VaR based on the actual probability distribution will provide a vital means of assessing better portfolio management.

## 6. Conflict of Interest

We certify that there is no conflict of interest with any financial organization regarding the material discussed in the manuscript.

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